# Links Between Children's Understanding of Multiplication and Solution Strategies For Division

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This paper reports on strategies 26 Year 3 students used to solve a range of division word problems in a one-to-one interview, following the participation of half of the group in a teaching experiment. The focus here is on the strategies used by case study students to solve *equivalent groups* and *times as many* division tasks. Results suggest that young children are capable of solving complex division problems given experience with a range of semantic structures for multiplication and division.

Studies on children's solutions to multiplication and division problems indicate that children as young as kindergarten age can solve a variety of problems by combining direct modelling with counting and grouping skills, and with strategies based on addition and subtraction (Anghileri, 1989; Bryant, 1997; Carpenter, Ansell, Franke, Fennema, & Weisbeck, 1993; Kouba, 1989; Mulligan, 1992). While this may be true, it is a widely held belief that multiplication and division are conceptually complex both in terms of the range of semantic structures (Anghileri, 1989; Kouba, 1989) and conceptual understanding (Steffe, 1988). To illustrate this point, consider solving 4x3. In using additive thinking, a child adds four collections of 3 (perceived as three ones) together. This involves only one level of abstraction, as the child makes inclusion relations on only one level. Multiplicative thinking on the other hand involves making two kinds of relations:"(a) the many-to-one correspondence between the three units of one and the one unit of three; and (b) the composition of inclusion relations on more than one level. Making three units of one into one unit of three is an abstraction at a higher level than thinking only of units of one" (Clark & Kamii 1996, pp. 42-43). Therefore to understand multiplication and division a child needs to "coordinate a number of equal sized groups and recognise the overall pattern of composites (view a collection or group of individual items as one thing), such as 'three sixes'"(Sullivan, Clarke, Cheeseman, & Mulligan 2001, p. 234). Sullivan et al. (2001) and Killion and Steffe (2002) suggested that the acquisition of an equal-grouping (composite) structure is at the core of multiplicative thinking.

To understand division requires more than knowledge of sharing out a collection equally; it requires an awareness of the relationship between the divisor and the quotient (Bryant, 1997). Knowledge of amounts each receives varies according to the number of recipients is not a consideration by many young children when sharing. Bryant maintained that a young child may be able to share using one to one correspondence but is unlikely to have an understanding of this relationship.

Fischbein, Deri, Nello, and Merino (1985) proposed two aspects of division: partitive and quotitive. In partition division (commonly referred to as the sharing aspect) the number of subsets is known and the size of the subset is unknown, whereas in quotition division (otherwise known as measurement division), the size of the subset is known and the number of subsets is unknown. Partition division has traditionally been taught before quotition (or measurement) division because the sharing aspect was considered to relate much more to a child's everyday life (Bryant, 1997; Haylock & Cockburn, 1997). However, Brown (1992) found that children in grade 2 performed better on quotition problems and tended to solve partitive problems using grouping strategies, rather than sharing strategies. Other research indicated that the sharing aspect of division is limited and of less relevance in the long term than quotition division (Correa, Nunes, & Bryant, 1998; Haylock & Cockburn, 1997).

Over the past decade, a number of studies have focused on children's solution strategies to multiplication and division (Anghileri, 1989; Brown, 1992; Kouba, 1989; Mulligan, 1992; Mulligan & Mitchelmore, 1997; Oliver, Murray, & Human, 1991; Steffe, 1988). These studies have provided evidence that children's solution strategies begin generally with direct modelling and unitary counting, progress to skip counting, double counting, repeated addition or subtraction, then to the use of known multiplication or division facts, commutativity and derived facts. As they progress, they are moving from relying on direct modelling to solve problems to partial modelling through to developing multiplicative thinking, at which point they are operating on problems abstractly. Prior to this point they are unable to form composites or integrate the composite structure with their counting strategies. Mulligan and Mitchelmore (1997) found that children build up a sequence of increasingly efficient intuitive models derived from previous ones, and rather than switch from one model to the next, they develop an increasing range of models to draw upon when solving a problem. Kouba (1989) found children used two intuitive strategies when solving quotition problems: either repeated subtraction or repeatedly building (double counting and counting in multiples). For partitive division, children drew on three intuitive strategies: sharing by dealing out by ones until the dividend was exhausted; sharing by repeatedly taking away; and sharing by repeatedly building up. Kouba (1989) suggested the need for further research on whether individual children have more than one model for multiplication or division, and whether they consistently employ similar models for partitive and quotitive division.

Mulligan and Mitchelmore (1997) used four categories listed by Greer (1992) namely: equivalent groups, multiplicative comparison, rectangular arrays and Cartesian product, in their study, Anghileri (1989) on the other hand examined children's responses and strategies to these six categories: equal groups, allocation/ rate, array, number line, comparison (times as many) and Cartesian product. In some studies these categories of multiplication situations are referred to as semantic structures (Kouba 1989; Mulligan & Mitchelmore, 1997; Schmidt & Weiser, 1995). Kouba identified two semantic factors specifically related to one-step multiplication and division word problems, which may influence children's solution strategies. The first relates to the differences in the interpretation of the quantities. For instance the interpretation of 3 in each of the following: equal groups (e.g., 3 cherries per plate, given 4 plates); comparison problems (e.g., 3 times as many); Cartesian product (e.g., possible combinations with 3 shirts and 2 ties), may prompt quite different solution strategies. The other semantic factor relates to the quantity that serves as the unknown in a problem. In partition division (number of subsets in each set is unknown) whereas in measurement division (number of sets is unknown). For example, 12 divided by 3 interpreted as a partition problem translates to a situation such as, 12 lollies shared between 3 people how many each? Interpreted, as a quotition problem would be: 12 lollies in bags of 3, how many bags?

The degree to which children's solution strategies may be influenced by the semantic structure of a problem or the quantities used, is inconclusive from the research (Anghileri, 2001; Clarke & Kamii, 1996; Mulligan, 1992; Mulligan & Mitchelmore, 1997). This paper reports on one aspect of a larger study that investigated the effects of providing students with a broad range of multiplication and division word problems based on different semantic structures, on their developing understanding of multiplication and division. The particular aspect being reported on in this paper focuses on *equivalent groups* and *times as many* word problems for both partitive and quotitive situations. The research questions that guided this aspect of the study are (a) Do children think flexibly about division, in particular 'times as many' using multiplicative thinking? (b) To what extend do children use their knowledge of multiplication and intuitive strategies in solving division problems?

## Methodology

The study was conducted from March to November 2007, and involved students aged eight and nine years from two grade 3 classes of two primary schools in a middle class suburb of Melbourne. One grade (EG) was part of a teaching experiment (TE); the other was used as a control group (CG). The TE occurred in two 12-day blocks, the first in May with the focus on multiplication, and the second in October when the focus was on division. The selection of the time frames was governed by the schools' schedules and the availability of the teachers. During these periods, the researcher and classroom teacher worked collaboratively.

The researcher planned the learning experiences each day in response to insights gained from the children's performance and strategies used. The teacher and researcher met for 30 minutes prior to each lesson and debriefed at the end of each lesson. The teaching approach involved a problem or question being posed and the students discussing possible strategies or methods for solving it. Word problems and open-ended tasks were the main context with some use of games. While the students were working the teacher and researcher roved and questioned them about the strategies they were using and their thinking. Often students were challenged to think of other ways they could solve the tasks and how they might check if the solution was correct.

*Participants*. While all 27 children in the EG were part of the TE, only 13 were selected as case studies from each grade, using a maximum variation sampling strategy (Patton, 2002). This enabled the researcher to gain a cross-section of each class according to their mathematical achievement. Prior to the TE, both grades were interviewed using the counting, addition and subtraction, multiplication and division domains of the Early Numeracy Research Project Interview (ENRP, Clarke, Cheeseman, Gervasoni, Gronn, Horne, McDonough, Montgomery, Roche, Sullivan, Clarke, & Rowley, 2002). These data were coded using a research-based framework of growth points, to identify the growth points reached by the students, and then students were ranked. The group was then divided into three bands and four children from the top and bottom band were chosen and five from the middle band. The classroom teacher in the TE had more than 20 years' teaching experience; the CG teacher was in her second year of teaching.

*Instruments*. The main sources of data collection were interviews. Two one-to-one, task-based interviews (Goldin, 1997) were used to probe and gain insights into students' understanding of multiplicative structures and strategies used in solving multiplication and division problems. There were three levels of questions for each of the following semantic structures: equivalent groups, allocation/rate, arrays, times as many, identified by Anghileri (1989) and Greer (1992). For each question, there were three levels of difficulty (rated by the researcher as easy, medium or challenge from pilot testing).

Multiplication word problems can be written as division problems (Greer, 1992; Mulligan & Mitchelmore 1997). The division interview consisted of 10 division word problems devised using the multiplication questions from the earlier interview. Each category included both a partitive (sharing) and quotitive (measurement) question to identify whether there was a relationship between the strategies students chose and the division type. Table 1 lists the questions chosen for discussion in this paper, categorising each as times as many or equivalent groups, noting the aspect of division (partition or quotation) and the rated level of difficulty. These were selected to show the contrasting strategies used for two quite different structures and because times as many is considered more difficult than the more commonly used equivalent groups (Haylock & Cockburn, 1997). In most instances, the contexts were the same for each problem type and the numbers chosen varied according to the level of difficulty.

**Table 1**

*Division Word Problems Used in the Study*

| Semantic structure | Aspect of division | Level of difficulty | Problem |
|---|---|---|---|
| **Equivalent groups** | Partition | Easy | I have 12 cherries to share equally onto 3 plates. How many cherries will I put on each plate? |
| | | Medium | I have 18 cherries to share equally onto 3 plates. How many cherries will I put on each plate? |
| | | Challenge | I have 48 cherries to share equally onto 3 plates. How many cherries will I put on each plate? |
| | Quotition | Easy | There are 12 children in the class. Three children sit at each table. How many tables are there? |
| | | Medium | There are 24 children in the class. Four children sit at each table. How many tables are there? |
| | | Challenge | 72 children compete in a sports carnival. Four children are in each event. How many events are there? |
| **Times as many** | Partition | Easy | Sam read 20 books during the read-athon, which was 4 times as many as Jack. How many books did Jack read? |
| | | Medium | Sam read 36 books during the read-athon, which was 4 times as many as Jack. How many books did Jack read? |
| | | Challenge | Sam read 72 books during the read-athon, which was 4 times as many as Jack. How many books did Jack read? |
| | Quotition | Easy | The Phoenix scored 18 goals in a netball match. The Kestrels scored 6 goals. How many times as many goals did the Phoenix score? |
| | | Medium | The Phoenix scored 28 goals in a netball match. The Kestrels scored 7 goals. How many times as many goals did the Phoenix score? |
| | | Challenge | The Phoenix scored 48 goals in a netball match. The Kestrels scored 16 goals. How many times many goals did the Phoenix score? |

## *Interview Approach*

The case study students in both schools were interviewed using these instruments three weeks after each twelve-day classroom intervention. The multiplication interview was administered to both cohorts in July and the division interview only in November. Each interview was audiotaped and took approximately 30 to 40 minutes, depending on the complexity of the student's explanation of the strategies used. Responses were recorded and any written responses retained. Students had the option of choosing the level of difficulty. Each question was presented orally, and paper, pencils and tiles were available for students to use at any time. If a student chose a difficult task and found it too challenging, there was an option for the student to choose an easier task.

## *Method of Analysis*

Initially, the author coded the students' responses as correct, incorrect, or non-attempt as well as the level of abstractness of solution strategies, drawing upon the categories of earlier studies (Anghileri, 2001; Kouba, 1989; Mulligan, 1998; Mulligan & Mitchelmore, 1997). Those chosen, listed and defined in Table 2 according to the level of abstraction, include direct and partial modelling, building up, repeated subtraction, doubling and halving, multiplicative calculation and wholistic thinking. For the purpose of this paper, the term abstractness refers to a student's ability to solve a problem mentally without the use of any physical objects (including fingers), drawings or tally marks. Where a student solved two tasks for the one question (easy and hard), the code for the more sophisticated strategy was recorded only. If a student used a strategy that reflected lack of understanding of the task, this was coded as an unclear strategy.

**Table 2**

*Solution Strategies for Whole Number Division Problems*

| Strategy | Definition |
|---|---|
| Unclear | Strategy reflects lack of understanding of task, or is unrelated to task. |
| Direct modelling | Uses sharing or one to many grouping with materials, fingers or drawings and calculates total by skip or additive counting. |
| Partial modelling | Partially models situation with concrete materials, or drawings using sharing or one to many grouping. Consistently uses skip or double counting to find the total. |
| Building up | Skip counts using the divisor up to the dividend. May use fingers to keep track of number counted. Records a number sentence in symbolic form. |
| Repeated Subtraction | Repeatedly taking away a specific number from the dividend until reaches zero, or skip counts back in multiples of the divisor from the dividend. Partial modelling in some instances. Records a number sentence in symbolic form. |
| Doubling and Halving | Derives solution using doubling or halving and estimation, attending to the divisor and dividend. Recognises multiplication and division as inverse operations. Records a number sentence in symbolic form. |
| Multiplicative Calculation | Automatically recalls known multiplication or division facts, or derives easily known multiplication and division facts, recognises multiplication and division as inverse operations. Records a number sentence in symbolic form. |
| Wholistic Thinking | Treats the numbers as wholes—partitions numbers using distributive property, chunking, and or use of estimation. |

Giving the students a choice and the need to identify the particular strategies used for both partitive and quotitive division tasks added to the level of complexity in presenting the data, as reflected in the tables and figures in the following section.

## Results and Discussion

Table 3 shows the comparison of results for both grades on equivalent groups and times as many partitive and quotitive whole number division problems. The responses for all 13 EG case study students were correct for both equal group tasks, and only one student in the CG response made an error. The number of correct responses varied on the times as many tasks. As indicated in Table 3, many more students in the CG gave incorrect responses for both the partitive and quotitive tasks, compared to the EG students. Only two students from the CG gave correct responses for the challenging tasks in the TMQ item during the interview, compared to nine students in the EG. Indeed, 12 EG students were able to give correct responses to items in the TMQ category compared with 3 CG students. This may be attributed to the fact that the students in the CG had little experience with such tasks prior to the interview, as their classroom learning sequence on division focused on equal group partition and quotition using direct modelling with materials leading to symbolic recording.

**Table 3**

*Comparison of Correct Responses for Whole Number Division Problems for Both Grades*

| Grade | Task | EGP | EGQ | TMP | TMQ |
|---|---|---|---|---|---|
| Experimental | Easy | | | | |
| *(n=13)* | Medium | 4 | 3 | 3 | 3 |
| | Challenging | 9 | 10 | 8 | 9 |
| Control | Easy | | | 1 | |
| *(n=13)* | Medium | 20 | 6 | 1 | 1 |
| | Challenging | 3 | 6 | 3 | 2 |

*Note.* EGP Equal group partition, TMP Times as many partition, EGQ Equal group quotation TMQ Times as many quotition

Table 4 shows the frequency of strategies by each grade on the different whole number division tasks. For space reasons, the strategies are not shown for each level of difficulty.

**Table 4**

*Distribution of Strategies Used for Times as Many and Equivalent Groups Tasks for Both Grades (Experimental (EXP) and Control (CON))*

| Grade | Task type | Unclear strategy | Direct modelling | Partial modelling | Building up | Repeated subtraction | Doubleing or halivng | Miltiplicative Calculation | Wholistic thinking |
|---|---|---|---|---|---|---|---|---|---|
| | EGP | | | 2 | 2 | | 1 | 6 | 2 |
| EXP | EGQ | | | 1 | 2 | | 3 | 6 | 1 |
| | TMP | | | | 2 | 2 | 4 | 3 | 2 |
| *(n=13)* | TMQ | | | | 3 | 2 | 3 | 3 | 2 |
| | EGP | | 3 | 5 | 1 | | | 3 | 1 |
| CON | EGQ | | 3 | 1 | 6 | | | 3 | |
| | TMP | 1 | 1 | 2 | 1 | 5 | 1 | | 2 |
| *(n=13)* | TMQ | 2 | 5 | 1 | 3 | | | 1 | 1 |

Multiplicative calculation was the preferred strategy of the EG (six students used this for the equivalent groups tasks), whereas the CG students used a wider range of strategies. In focusing on the times as many tasks, EG students predominantly used multiplicative strategies (wholistic thinking-four, multiplicative calculation-six, or doubling and halving-seven) and gave correct responses, whereas students in the CG predominantly used some form of modelling or a strategy that reflected little or no understanding of the task (building up-three, partial modelling-three, direct modelling-six, unclear strategy-three) with only eight correct responses (as indicated in Table 3) over the two times as many tasks. Over the four task types, multiplicative calculation was used most frequently (15), doubling and halving (eleven), building up (11) and wholistic thinking (8), in the EG. By contrast, direct modelling was used most frequently (12) in the CG, building up (11), partial modelling (8), multiplicative calculation (7).

Students in the CG rarely drew on their knowledge of multiplication when solving division tasks, and when they did, it was in the form of skip counting. They tended to use sharing in the partition tasks and repeated subtraction or count back, in the times as many tasks (as indicated by Kouba 1989). Students in the EG, on the other hand, tended to draw on their knowledge of multiplication facts as a starting point, rather than repeated subtraction or count back. They were able to record a division number sentence and explain the relationship between the numbers in both a multiplication and division number sentence. Many students in the EG during the interview commented that division is easier than multiplication because "you know how many you have to start with and how many groups or how many to divide it between".

The following excerpts from the students exemplify some of these strategies. To solve the equivalent groups quotitive problem, Bianca used known multiplication facts and doubling and Jack used doubling.

> Bianca: I started with 12x4 then doubled it to get 24x4 and that's 96, but that's too much. So I took away 20 from 96 and that gave me 76, and that's 19x4 but I still need to take away another 4 so it would be 18 events or 72 ÷ 4 =18.
>
> Jack: 4 children in each event and 72 children is 18 events, 'cause 4 plus 4 is 8 (that's 2), 8 plus 8 is 16 (that's 4), 16 plus 16 is 32 (that's 8), 32 and 32 is 64 (that's 16), 64 and 2 fours more is 72, so it is 18 fours. The number sentence would be 72 ÷ 4 = 18.

To solve the times as many partitive problem, Samantha used wholistic thinking by splitting the product and using the distributive property. Jack on the other hand used halving and his place value and fraction knowledge.

Samantha: I need to think of 4 times something equals 72. 72 take away 40 (which is 4 times 10) is 32, and 32 is 4x8. 4x20 is 80, but that's too much. I know 6x12 equals 72, but 12 would be 6 times not 4 so it can't be that. I need to take 8 off 80 to get 72 so it would be 18, because 4x10 is 40 and 4x8 is 32 and together that's 72. So Sam read 18 books, 72 ÷ 4 = 18.

Jack: Half of 7 is 3 and a half, so half of 70 would be 35 and half of 72 would be 36 because half of 2 is 1. Half of 3 is one and a half, so half of 30 is 15 and half of 6 is 3 so it is 18 books. 72 ÷ 4 = 18 because you halve 72 two times, which is the same as saying 18, four times.

The numbers used in the equal groups partitive and times as many quotitive tasks were the same, as were the numbers in the equal groups quotitive and times as many partitive, but none of the children recognised this. These students solved times as many tasks more efficiently (and in less time) than the equal groups tasks, which was surprising. In each instance, they were clearly drawing on their problem solving skills and knowledge of number as they were thinking about the problem. Jack consistently used doubling and halving; the others used multiplication.

## Conclusion

The results suggest that young children are capable of solving complex division problems when provided with a problem solving learning environment that encourages them to draw on their intuitive thinking strategies and knowledge of multiplication. Given an opportunity to experience a range of semantic structures for multiplication provides a solid basis for children's developing understanding of division. This finding resonates with the work of Mulligan and Mitchelmore, (1997) but this study adds to the body of knowledge an intensive look at the *times as many* structure of multiplication as applied to both partitive and quotitive aspects of division.

Children need a variety of experiences with different semantic structures and contexts to understand fully the operations of multiplication and division. These experiences need to include both partitive and quotitive aspects of division in the early years, using contexts that relate to children's everyday lives. This implies allowing students to draw on their own intuitive strategies to solve both partitive and quotitive division word problems, prior to formal teaching. Placing emphasis on the relationship between multiplication and division and the language associated with both operations before any use of symbols or formal recording needs to be a priority. The study also provides an argument to support delaying the introduction of any formal algorithm for division until children have a sound conceptual understanding of division, and are confident in solving division tasks beyond the multiplication fact range mentally. A possible question for further research: Do children retain the richness of mental strategies when taught the formal written algorithms?

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